INVESTIGATION OF NON-STEADY-STATE CENTRALLY SYMMETRIC FILTRATION OF A LIQUID IN CLOSED HETEROGENEOUS MEDIA

S. N. Bagir-zade, G. P. Guseinov, and A. G. Kerimov

UDC 622.276.031:532:5

This paper is the final stage in a study of the properties of the non-steady-state filtration of a homogeneous liquid towards a central well with a hemispherical end face in heterogeneous media with a dual porosity, consisting of hemispherical regions set one inside the other with different values of the medium parameters [1, 2]. Exact solutions are found for the problems of the decrease in seam pressure as a function of time and distance, as well as of the time change in output of a well with a hemispherical end face operating at a fixed flow rate, or at a fixed end face pressure, respectively, in closed heterogeneous media. The effect of the magnitudes of the heterogeneous media parameters on the change in their production process indicators is established on the basis of numerical calculations.

1. Formulation of the Problem

An analysis of the production process in petroleum deposits with fissured-porous type collectors, modeled as heterogeneous media with dual porosity, shows that the majority of production wells reveal the productive capacity of the seams at an inconsiderable depth. Thus, the greater the capacity of the seam and the less the depth of well penetration into the seam, the more closely will the liquid flow in the immediate neighborhood of the well approximate to a radial flow, i.e., centrally symmetric.

Non-steady-state centrally symmetric problems of the filtration of a homogeneous liquid in fissured-porous media are investigated in [1, 2] when the filtration region is either unlimited in extent, or the external hemispherical surface is a surface of equal pressure.

We shall assume that around the well of radius R_W there is a hemispherical region $R_W \le r \le R_0$ with one penetrability of the fissure system, while beyond it $R_0 \le r \le R_1$ the penetrability of the fissures has another magnitude. As distinct from [1, 2] the external contour $r = R_1$ is impenetrable. We must determine the process of pressure decrease at an arbitrary point of the hemispherical media set one inside the other, and the output of a well with a hemispherical end face in the production process. The problem reduces to integrating the system of equations [3, 4].

$$\frac{\partial^2\psi_i^{(2)}}{\partial\xi^2} + \frac{2}{\xi}\frac{\partial\psi_i^{(2)}}{\partial\xi} - \frac{1-\omega}{k_i}\frac{\partial\psi_i^{(1)}}{\partial\tau} = \frac{\omega}{k_i}\frac{\partial\psi_i^{(2)}}{\partial\tau}; \qquad (1.1)$$

$$\frac{\partial\psi_i^{(1)}}{\partial\tau} + \Lambda\psi_i^{(1)} = \Lambda\psi_i^{(2)} \quad (i = 1, 2)$$

for zero initial conditions and the following boundary conditions (the superscript 2 is omitted):

$$\psi_1(\xi_0;\tau) = \psi_2(\xi_0;\tau);\ \partial\psi_1(\xi_0;\tau)/\partial\xi = k_0\partial\psi_2(\xi_0;\tau)/\partial\xi; \qquad (1.2)$$

$$\partial\psi_2(\xi_1;\tau)/\partial\xi = 0;\ \xi_0 = R_0/R_W,\ \xi_1 = R_1/R_W.$$

Here the following symbols have been introduced:

Baku. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 98-104, May-June, 1976. Original article submitted May 16, 1975.

$$\psi_i^{(j)}(\xi;\tau)=\frac{2\pi k_1^{(2)}R_w}{\mu q_0}\left[p_0-p_i^{(j)}(\xi;\tau)\right];\quad \xi=r/R_w;$$

$$\tau=k_1^{(2)}t\mu^{-1}R_w^{-2}(\beta_1+\beta_2)^{-1};\quad \Lambda=\lambda(1-\omega)^{-1};\quad \lambda=\alpha R_w^2\frac{k_i^{(1)}}{k_i^{(2)}};\tag{1.3}$$

$$\omega=\beta_2(\beta_1+\beta_2)^{-1};\quad k_i=\begin{cases}1, & i=1\\ k_0=k_2^{(2)}/k_1^{(2)}, & i=2;\end{cases}$$

α is the coefficient determining the exchange of liquid between the systems of blocks and fissures in the medium, p_0 and $p(\xi;\tau)$ are the initial pressure and the pressure at any subsequent time, k and μ are the penetrability coefficient of the porous medium and the coefficient of dynamic viscosity of the filtering liquid, β_1 and β_2 are the elasticity coefficients of the porous blocks and fissures of the medium, R_w is the well radius, ω is the parameter of fissure capacity, λ is a parameter which characterizes the degree of difficulty with which fluid is exchanged between the systems of blocks and fissures of the heterogeneous medium, and r and t are the radial coordinate and time. The superscripts in the functions of pressure and seam parameters refer to the system of blocks (1) and fissures (2) of the medium, and the subscripts to the hemispherical regions of the seam system set one inside the other.

Solutions of the problem are given below for different well production patterns (depending on the conditions for $\xi=1$).

2. Determination of the Pressure Field for Closed Fissured-Porous Seams with a Fixed Well Output

The conditions at the well are written in the form

$$\lim_{\xi\to 1}\left[\xi^2\partial\psi_1(\xi;\tau)/\partial\xi\right]=-q(\tau).\tag{2.1}$$

On solving the system (1.1) with the conditions (1.2), (2.1) in a manner similar to that described in [2], we obtain the formula for pressure reduction

$$\psi_i(\xi;\tau)=f_i(\xi;\tau)+\frac{\omega}{\Lambda\xi}\sum_{m=1}^{\infty}\frac{\Phi(p_m;\tau)}{p_m^2\delta(p_m)}\frac{U_i(\xi_1,\xi,p_m)}{W_1(\xi_0,\xi_1,p_m)}\qquad (i=1,2),\tag{2.2}$$

$$U_1(\xi_1,\xi,x)=\left(\xi_1,x-\frac{\varepsilon_-k_0}{\xi_0 x}\right)\varepsilon_+\cos(X_1-\xi)x-\left(\frac{\varepsilon_+\xi_1}{\xi_0}+\sqrt{k_0}\right)\varepsilon_-\sin(X_1-\xi)x-$$
$$-\frac{\xi_1+\xi_0\sqrt{k_0}}{\xi_0}\varepsilon_-\varepsilon_+\sqrt{k_0}\sin(X_2+\xi)x+\left(\xi_1 x+\frac{\varepsilon_+k_0}{\xi_0 x}\right)\varepsilon_-\cos(X_2+\xi)x;$$

$$U_2(\xi_1,\xi,x)=\frac{\xi_1 x}{\sqrt{k_0}}\cos\frac{\xi_1-\xi}{\sqrt{k_0}}x-\sin\frac{\xi_1-\xi}{\sqrt{k_0}}x;$$

$$W_1(\xi_0,\xi_1,p_m)=\lim_{x\to -p_m^2}\frac{\partial}{\partial x}V_1(\xi_0,\xi_1,x);$$

$$V_1(\xi_0,\xi_1,x)=\left[\frac{\xi_0-\sqrt{k_0}}{\xi_0 x}-\varepsilon_+\sqrt{x}\left(\frac{\varepsilon_-\xi_1}{\xi_0}\sqrt{k_0}-\xi_1+\sqrt{k_0}\right)\right]\operatorname{ch}(X_1-1)\sqrt{x}+$$
$$+\left[\varepsilon_-\xi_1 x+\left(\frac{\sqrt{k_0}-\xi_1}{\xi_0}\varepsilon_--1\right)\varepsilon_+\sqrt{k_0}\right]\operatorname{sh}(X_1-1)\sqrt{x}+$$
$$+\left[\xi_1\sqrt{x}-\left(\frac{\xi_1\sqrt{x}}{\xi_0\sqrt{k_0}}+\frac{1}{\xi_0\sqrt{x}}-\sqrt{x}\right)\varepsilon_+k_0\right]\varepsilon_-\operatorname{ch}(X_2+1)\sqrt{x}+$$
$$+\left[\xi_1 x+\left(\frac{\xi_1}{\xi_0 x}+\frac{1}{\xi_0}-1\right)\varepsilon_+k_0\right]\varepsilon_-\operatorname{sh}(X_2+1)\sqrt{x};$$

$$X_{1,2}=(\xi_1-\xi_0)/\sqrt{k_0}\pm\xi_0,\ \varepsilon_\pm=\left(1\pm\sqrt{k_0}\right)/\sqrt{k_0},\ \delta(x)=\left[(\Lambda+x^2)^2-4\Lambda\omega x^2\right]^{1/2};$$

$$\Phi(x;\tau)=s_2(x)\left[s_1(x)+\Lambda\right]\exp\left[s_1(x)\tau\right]-s_1(x)\left[s_2(x)+\Lambda\right]\exp\left[s_2(x)\tau\right];$$

$$f_i(\xi;\tau)=\begin{cases}(Ac-BD)/\xi A^2+(B/\xi A)\,\Omega(\Lambda;\tau), & i=1\\ \dfrac{\xi_0}{k_0^{3/2}\xi}\left[\dfrac{\xi}{A}\Omega(\Lambda;\tau)-\dfrac{\xi D}{A^2}+\dfrac{\xi+2\xi_1}{6k_0\xi}(\xi_1-\xi)^2\right], & i=2,\end{cases}$$

$$\Omega(\Lambda;\tau)=\tau+\left[(1-\omega)/\Lambda\right]\left[1-\omega(1-\omega)\exp(-\Lambda\tau/\omega)\right],\quad B=\xi_0\xi/k_0^{3/2};$$

$$A=\frac{\xi_0}{3k_0^{3/2}}\left[\left(\xi_1^2+\xi_0^2+\xi_0\xi_1\right)(\xi_1-\xi_0)+\left(1+\xi_0+\xi_0^2\right)(\xi_0-1)\right],$$

$$c=\frac{1}{3k_0^{3/2}}\left[\left(\xi_1^2+\xi_0^2+\xi_1\xi_0\right)(\xi_1-\xi_0)(\xi_1-\xi)+(\xi_0-\xi)^2\xi_0(\xi+2\xi_0)+\frac{\xi}{2k_0}(\xi_1-\xi_0)^2(\xi_0-2\xi_1)\right],$$

$$D=\frac{1}{30k_0}\left[\frac{\xi_0}{k_0^{3/2}}(\xi_1-\xi_0)^3(\xi_1^2+\xi_0^2+3\xi_1\xi_0)+\xi_0(\xi_0-1)^3(\xi_0^2+3\xi_0+1)+\right.$$

$$\left.+\frac{5}{3\sqrt{k_0}}(\xi_1-\xi_0)(\xi_0-1)^2(\xi_0+2)(\xi_1^2+\xi_0^2+\xi_1\xi_0)+\frac{5}{3k_0^{3/2}}(\xi_1-\xi_0)^2(\xi_0-1)(\xi_0+2\xi_1)(\xi_0^2+\xi_0+1)\right];$$

$$s_{1,2}(x)=(1/2\omega)[-(\Lambda+x^2)^2\pm\delta(x)]. \tag{2.3}$$

The values p_m^2 $(m=1,2,\ldots)$ arethe roots of the equation

$$V_1(\xi_0,\ \xi_1,\ p)=0. \tag{2.4}$$

The resulting solution Eq. (2.2) describes the process of pressure reduction at an arbitrary point in the fissure system of a fissured-porous seam of composite penetrability with a well having a hemispherical end face and for a constant output of liquid.

For the fissure system of a fissured-porous seam of uniform penetrability the corresponding formula for pressure reduction is obtained from the formula given above by setting $k_0=1$.

3. Determination of Well Output in the Production from a Closed Fissured-Porous Seam for a Fixed End Face Pressure

It is assumed that a constant pressure p_w is maintained during the entire production process at the hemispherical surface of a centrally placed production well in the fissure system of a fissured-porous seam of composite penetrability. The normalized pressure reduction function in the fissure system

$$\psi_i(\xi;\tau)=[p_0-p_i(\xi;\tau)](p_0-p_c)^{-1}\qquad(i=1,2)$$

satisfies the conditions (1.2) and the condition

$$\lim_{\xi\to 1}\psi_1(\xi;\tau)=1.$$

Proceeding in the manner outlined above, the well output

$$q(\tau)=-(\partial/\partial\xi)[\xi\psi_1(\xi;\tau)]_{\xi=1}$$

can be described by the following analytical expression:

$$q(\tau)=\frac{2\omega}{\Lambda}\sum_{m=1}^{\infty}\frac{\Phi(p_m;\tau)}{p_m^2\delta(p_m)}\frac{W_3(\xi_0,\xi_1,p_m)}{W_2(\xi_0,\xi_1,p_m)}, \tag{3.1}$$

where

$$W_2(\xi_0,\xi_1,x)=\left[\frac{\varepsilon_-k_0}{\xi_0x^2}+(X_1-1)\frac{\varepsilon_-\xi_1-\xi_0}{\xi_0}\sqrt{k_0}\right]\frac{\varepsilon_+}{x}\cos(X_1-1)x+$$

$$+\left(\frac{\varepsilon_-k_0}{\xi_0x^2}+\xi_1\right)\varepsilon_+(X_1-1)\sin(X_1-1)x-\left(\frac{\varepsilon_-k_0}{\xi_0x^2}-\xi_1\right)\varepsilon_-(X_2+1)\times$$

$$\times\sin(X_2+1)x+\left[\xi_1-\frac{\varepsilon_+k_0}{\xi_0x^2}+\left(\frac{\xi_1}{\xi_0}-\sqrt{k_0}\right)(X_2+1)\right]\frac{\varepsilon_-}{x}\cos(X_2+1)x;$$

$$W_3(\xi_0,\xi_1,x)=\left[(\sqrt{k_0}+\xi_1x^2)\varepsilon_++\frac{\xi_1+\varepsilon_+k_0}{\xi_0}\varepsilon_-\right]\sin(X_1-1)x+$$

$$+\left[\frac{\xi_0+\varepsilon_-\xi_1}{\xi_0}\sqrt{k_0}-\xi_1+\frac{\varepsilon_-k_0}{\xi_0x^2}\right]\varepsilon_+x\cos(X_1-1)x+$$

$$+\left(\varepsilon_+k_0+\xi_1x^2-\frac{\xi_1+\sqrt{k_0}}{\xi_0}\varepsilon_+\sqrt{k_0}\right)\varepsilon_-\sin(X_2+1)x+$$

$$+\left[\left(\frac{\xi_1x^2-\sqrt{k_0}}{\xi_0x^2}-\sqrt{k_0}\right)\varepsilon_+\sqrt{k_0}-\xi_1\right]\varepsilon_-x\cos(X_2+1x).$$

Here p_m $(m=1,\ 2,\ldots)$ are the roots of the equation

$$(\xi_0\xi_1p^2-\varepsilon_-k_0)\varepsilon_+\cos(X_1-1)p-(\varepsilon_-\xi_1+\varepsilon_+\xi_0\sqrt{k_0})p\sin(X_1-1)p+$$

$$+(\xi_0\xi_1p^2+\varepsilon_+k_0)\varepsilon_-\cos(X_2+1)p-(\xi_0\sqrt{k_0}-\xi_1)\varepsilon_-p\sin(X_2+1)p=0. \tag{3.2}$$

Formula (3.1) describes the change in time of output of a well with a hemispherical end face in the fissure system of a closed fissured-porous seam of composite penetrability.

TABLE 1

m	$10^4 p_m$ (2.4)	$\lambda=0{,}005$: $-10^4 s_1$	$\lambda=0{,}005$: $-10^2 s_2$	$\lambda=1$: $-10^4 s_1$	$\lambda=1$: $-10^2 s_2$
1	2045	54	209	1750	1298
2	6157	55	621	4057	1686
3	12508	55	1256	6038	2302
4	21126	55	2118	7458	3149
5	32244	56	3229	8427	4251
6	45703	56	4575	9083	5590
7	61581	56	6163	9538	7174
m	$10^4 p_m$ (3.2)	$\lambda=0{,}005$		$\lambda=0{,}01$	
1	603	35	8	27	14
2	5017	52	256	107	262
3	12126	53	744	110	749
4	15637	54	1475	110	1480
5	19141	55	2450	110	2455
6	22900	55	3669	111	3674
7	25152	56	5013	111	5014
8	30123	56	6644	111	6655
9	33814	56	8313	111	8351
10	37226	56	10022	111	10110

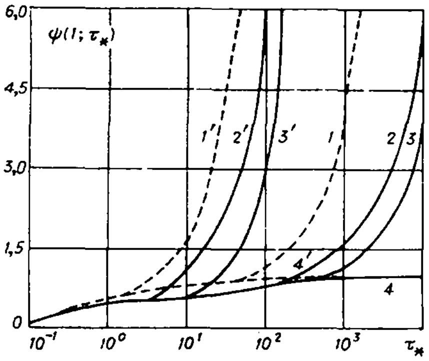

Fig. 1

If the fissure system of the plate is assumed to have uniform penetrability ($k_0=1$) the formula given above simplifies considerably, since the following relation then becomes valid:

$$W_3(\xi_0, \xi_1, p_m)/W_2(\xi_0, \xi_1, p_m)_{k_0=1} = \left[\left(1+\xi_1 p_m^2\right)\sin(\xi_1-1)\,p_m - p_m(\xi_1 - 1)\cos(\xi_1-1)\,p_m\right]\left[(1/p_m)\cos(\xi_1-1)\,p_m - \xi_1(\xi_1-1)\sin(\xi_1-1)\,p_m\right]^{-1}. \tag{3.3}$$

The well-known solution for this problem in the case of a homogeneous granular medium [15] can be obtained from Eq. (3.1).

4. Discussion of Results of the Calculations

In order to make numerical calculations from the basic formulas of the solutions to the problems, we must have numerical values of $s_1(p_m)$ and $s_2(p_m)$ as determined from Eq. (2.3) depending on the roots of Eqs. (2.4), (3.2). For this purpose a table was composed of various values of the parameter λ for $k_0=1$, $\omega=0.1$, and $\xi_1=10$. It follows from Table 1 that as m increases the values of $s_2(p_m)$ increase without limit, while the following relation becomes valid for the values of $s_1(p_m)$:

$$\lim_{m\to\infty} s_1(p_m) = -\Lambda.$$

Calculations were made from the data of Table 1, and the results are given in Fig. 1. The dashed lines correspond to the granular medium and are constructed from the data of [15] for $\xi_1=3$, 10 (curves 1' and 1, respectively).

Graphs of the time

$$\tau_* \equiv \tau/\omega = k^{(2)}t/\mu \dot{R}_w^2 \beta_2$$

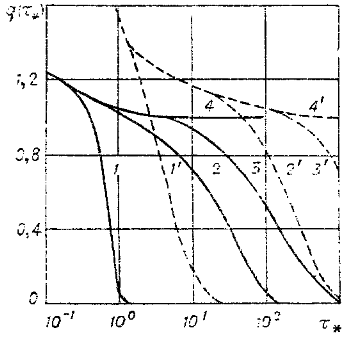

Fig. 2

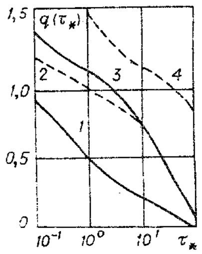

Fig. 3

variation of the pressure reduction functions $\psi(1, \tau_*)$ at the end face, constructed from Eq. (1.3) for hypothetical petroleum deposits with fissured-porous type collectors with $\xi_1 = 3; 10, \omega = 0.1$ and values of the parameter $\lambda = 0.005$; 1; 0.005 and 0.01 are given by the curves 2, 3, 3', and 2', respectively. The envelopes of these curves 4' and 4 correspond to spatially unbounded petroleum seams with granular and fissured-porous type collectors.

These calculations and an analysis of the formula for the pressure reduction process (for $k_0 = 1$) show that as the dimensionless time increases the formula in the fissured-porous medium differs from that in the granular medium by the quantity

$$(1-\omega)^2 \lambda^{-1} \left[(1/3)(\xi_1 - 1)^4 + 3\xi_1^2 + 2\xi_1(\xi_1 - 1)^2\right]^{-1}.$$

However, as the radius of the impenetrable outer boundary of the seam increases, the reduction in end-face pressure of a well with a hemispherical end face producing at a constant output is observed to stabilize with time, both in the granular as well as in the fissured-porous seams. This follows from a comparison of curves 1'-3' and 1-3 with the envelopes 4' and 4. A similar phenomenon is not characteristic for other forms of liquid flow, neither linear nor plane parallel [9-14].

Results of calculating the output of a well with a hemispherical end face in closed granular and fissured porous seams of homogeneous ($k_0 = 1$) penetrability from formulas (3.1), (3.3) are given in Fig. 2. The dashed curves correspond to a granular medium ($\omega = 1$, $\lambda = \infty$) and the continuous to fissured-porous media ($\omega = 0.1$, $\lambda = 0.01$). Curves 1, 1'; 2, 2'; 3 and 3' correspond to petroleum deposits with impenetrable outer boundaries with $\xi_1 = 3$, 10, and 20 units of the dimensionless radius. The envelopes of these curves 4' and 4 correspond to spatially unbounded seams with granular and fissured-porous type collectors. It follows from a comparison of curves 1-4 and 1'-4' that the output of wells for centrally symmetric filtration in fissured-porous media does not exceed the output of wells in granular media for a fixed production time.

This is explained by the fact that part of the volume in fissured-porous petroleum seams (depending on the magnitude of the parameter ω) contains inclusions of low pentrability which result in lower production outputs for a well with a hemispherical end face. In granular seams the whole volume has the same penetrability equal to the penetrability of the fissure system $k^{(2)}$ of a fissured-porous seam, and there are no inclusions of penetrability ($\omega = 1$). The functions $q(\tau_*)$ are given in Fig. 3 for various values of the parameters ω, λ [curve 1) $\omega = 0.1$, $\lambda = 0.005$; 2) $\omega = 0.1$, $\lambda = 0.01$; 3) $\omega = 0.5$, $\lambda = 0.005$; 4) $\omega = 1$, $\lambda = \infty$]. These curves enable a quantitative analysis to be made of the time change in production output of a well with a hemispherical end face in closed fissured-porous seams and for it to be compared with that in a granular medium. In fact, the data of Fig. 3 can be used to predict the further behavior of the preproduction process indicators for fissured-porous petroleum seams with known values of the parameters ω and λ.

The initial system of filtration equations (1.1) and also the solutions obtained enable us to conclude that in fissured-porous seams as the fissure capacity parameter ω approaches unity and as the quantity λ increases, the time variation of well output in fissured-porous media tends to assume the character of the variation in well output in a granular medium. The values of the functions in the computed formulas were taken from [16-18].

LITERATURE CITED

1. S. N. Bagir-zade and G. P. Guseinov, "Nonsteady-state filtration of a liquid in a fissured-porous seam to a well with a hemispherical end face," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1973).

2. S. N. Bagir-zade, G. P. Guseinov, and A. G. Kerimov, "Centrally symmetric filtration of a liquid in fissured-porous media with a hemispherical feed surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1975).
3. G. I. Barenblatt, Yu. P. Zheltov, and N. N. Kochina, "Fundamental concepts in the theory of filtration of homogeneous liquids in fractured rocks," Prikl. Mat. Mekh., 25, No. 5 (1960).
4. J. E. Warren and P. J. Root, "The behavior of naturally fractured reservoirs," Soc. Petrol. Eng. J., 3, No. 3 (1963).
5. M. A. Lavrent'ev and B. V. Shabat, Methods in the Theory of Functions of a Complex Variable [in Russian], Nauka, Moscow (1973).
6. A. F. Van Everdingen and W. Hurst, "The application of the Laplace transformation to flow problems in reservoirs," J. Petrol. Technol., No. 12 (1949).
7. R. I. Medvedskii, "Interrelation between the first and second filtration phases in a porous-fractured seam," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1969).
8. S. N. Bagir-zade and I. A. Nasrullaev, "Non-steady-state filtration of a liquid towards a rectilinear gallery in a two-layer fissured-porous seam," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1974).
9. S. N. Bagir-zade and G. P. Guseinov, "Non-steady-state filtration of a liquid in a fractured-porous seam," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1972).
10. G. P. Guseinov and S. N. Bagir-zade, "One-dimensional movement of a liquid in a fissured-porous medium," Tr. AzNIPI Neft', No. 24 (1972).
11. G. P. Guseinov and S. N. Bagir-zade, "Flow of liquid to a well partially penetrating an inhomogeneous fissured-porous seam for non-steady-state filtration," Azerb. Neft. Khoz., No. 8 (1971).
12. G. P. Guseinov, S. N. Bagir-zade, and A. G. Kerimov, "Flow of a liquid from a fissured-porous seam to a well with a constant end-face pressure," in: Questions in Production from Petroleum and Gas Deposits [in Russian], Azerbaidzhana, Baku (1974).
13. S. N. Bagir-zade, "Flow of a homogeneous liquid to a gallery in a fissured-porous seam," Azerb. Neft. Khoz., No. 7 (1972).
14. S. N. Bagir-zade, "Nonsteady-state flow of a homogeneous liquid to a gallery in a fissured-porous seam," Tr. AzNIPI Neft', No. 28 (1973).
15. A. F. Chatas, "Unsteady spherical flow in petroleum reservoirs," Soc. Petrol. Eng. J., 2, No. 2 (1966).
16. M. Barlow, Tables [Russian translation], Mir, Moscow (1965).
17. B. I. Segal and K. A. Semendyaev, Five-Figure Mathematical Tables [in Russian], Izd. Akad. Nauk SSSR, Moscow (1948).
18. K. Kayashi and L. Bark, Multifigure Tables of Circular, Hyperbolic, and Other Functions [in Russian], Izd. Akad. Nauk SSSR, Moscow (1965).